

## CLAIMS

I claim:

- 1           1.     An electronic voting system for use with a  
2 computerized network, comprising:  
3                 a plurality of voting computers coupled to the computerized  
4 network, wherein each voting computer provides an electronic encrypted  
5 ballot, wherein each electronic ballot is encrypted under a discrete log  
6 asymmetric encryption process using underlying groups  $Z_p$  or elliptic curve;  
7                 at least first, second and third authority computers coupled to  
8 the computerized network, wherein the first authority computer is  
9 configured to receive a series of electronic ballots corresponding to an  
10 aggregation of each of the electronic ballots received from the plurality of  
11 voting computers, and to apply a secret, one-way cryptographic  
12 transformation using at least a first secret key to anonymously shuffle the  
13 series of electronic ballots and produce a first shuffled series of ballots,  
14 wherein only the first authority computer knows a correspondence between  
15 the first series of shuffled ballots and the series of electronic ballots, and  
16 wherein the first authority computer is further configured to provide a first  
17 linear size, non-interactive proof of correctness for the first series of  
18 shuffled ballots based on a scaled iterated logarithmic multiplication proof;  
19                 wherein the second authority computer is configured to  
20 receive the first series of shuffled ballots, to apply the cryptographic  
21 transformation using at least a second secret key to anonymously shuffle the  
22 first series of shuffled ballots and produce a second series of shuffled  
23 ballots, wherein only the second authority computer knows a  
24 correspondence between the first series of shuffled ballots and the second  
25 series of shuffled ballots, and wherein the second authority computer is

26 further configured to provide a second linear size, non-interactive proof of  
27 correctness for the second series of shuffled ballots based on the scaled  
28 iterated logarithmic multiplication proof;

29 wherein the third authority computer is configured to receive  
30 the second series of shuffled ballots, to apply the cryptographic  
31 transformation using at least a third secret key to anonymously shuffle the  
32 second series of shuffled ballots and produce a third series of shuffled  
33 ballots, wherein only the third authority computer knows a correspondence  
34 between the third series of shuffled ballots and the second series of shuffled  
35 ballots, and wherein the third authority computer is further configured to  
36 provide a third linear size, non-interactive proof of correctness for the third  
37 series of shuffled ballots based on the scaled iterated logarithmic  
38 multiplication proof; and

39 a verification computer coupled to the computerized network,  
40 wherein the verification computer is configured to receive the proofs of  
41 correctness from the first, second and third authority computers and without  
42 interacting with the first, second and third authority computers, to verify a  
43 correctness of the shuffled ballots.

1 2. The system of claim 1, further comprising:

2 a server computer system coupled to the computerized  
3 network, wherein the server computer system is configured to: receive the  
4 plurality of electronic ballots from the plurality of voting computers; verify  
5 a proof of validity of each of the plurality of electronic ballots; form an  
6 encrypted tally of the votes from the plurality of electronic ballots; transmit  
7 the encrypted tally to the first, second and third authority computers;  
8 receive ballot decryption shares produced from at least two of the first,  
9 second and third authority computers; and compute a decrypted tally; and

10 at least one voting poll computer coupled to the computerized  
11 network and providing some of the plurality of electronic encrypted ballots  
12 to the server computer system.

1 3. The system of claim 1 wherein the first, second and  
2 third authority computers are configured to provide Chaum-Pedersen proofs  
3 for the first, second and third shuffles of the ballots, respectively, and  
4 wherein each of the first, second and third authority computers generate an  
5 initial challenge series, receive a challenge from at least one verification  
6 computer, and generate the cryptographic transformation based on an  
7 exponentiation of the initial and received challenges.

1 4. The system of claim 1 wherein the computerized  
2 network includes the World Wide Web, wherein each of the plurality of  
3 voting computers and first, second and third authority computers include a  
4 web browser program.

1 5. The system of claim 1 wherein the plurality of voter  
2 computers include at least one palm-sized computer, cell phone, wearable  
3 computer, interactive television terminal or Internet appliance.

1 6. A computer system for receiving a sequence of  
2 elements, comprising:

3 a server computer coupled to a computer network and  
4 configured to:

5 receive a sequence of electronic data elements  
6 representing individual data files,

7 apply a cryptographic transformation using at least a  
8 first secret key to anonymously permute the sequence of electronic  
9 data elements and produce a first shuffled sequence of electronic

10 data elements, wherein the server computer knows a correspondence  
 11 between the first shuffled sequence of electronic data elements and  
 12 the sequence of electronic data elements, and  
 13 generate a first linear size proof of correctness for the  
 14 first shuffled sequence of electronic data elements based on a scaled  
 15 iterated logarithmic multiplication proof.

1 7. The system of claim 6 wherein the received sequence  
 2 of electronic data elements are encrypted using  $Z_p$  or elliptic curve groups  
 3 using a key unknown to the server computer, and wherein the server  
 4 computer is further configured to:

5 receive a series of randomly generated values  $e_i$  from a  
 6 verifier computer;

7 secretly generate a series of values  $U_i$  based on a secret, one-  
 8 way cryptographic transformation that employs the received series of values  
 9  $e_i$  and secretly generated values

$$\overline{u_i}$$

10 permute the sequence of electronic data elements to produce  
 11 the first shuffled sequence of elements based on the series of values  $U_i$  and a  
 12 secret value  $d$ ; and

13 provide the values  $U_i$  and a series of proof values based on the  
 14 cryptographic transformation as a proof of knowledge that the server  
 15 computer has access to how the cryptographic transformation permuted the  
 16 sequence of electronic data elements to produce the first shuffled sequence  
 17 of elements without revealing the cryptographic transformation to the  
 18 verifier computer.

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1                   8.     The system of claim 6 wherein the server computer is  
2 further configured for:

3                   receiving a plurality of public keys from a corresponding  
4 plurality of individuals, wherein each of the plurality of individuals have a  
5 private key corresponding to one of the plurality of public keys;

6                   receiving a request for a certificate from one of the plurality  
7 of individuals having a one private key;

8                   providing at least a subset of the plurality of public keys to the  
9 requesting individual;

10                  receiving a shuffle of the plurality of public keys and a linear  
11 size proof of correctness for the shuffled public keys based on a scaled  
12 iterated logarithmic multiplication proof and a value corresponding to the  
13 one private key, wherein the value provides proof that the one individual  
14 has knowledge of the one private key without revealing the one private key;

15                  checking the proof of correctness;

16                  checking that the value is mathematically related to a one of  
17 the public keys that corresponds to the one private key;

18                  issuing a certificate to the one individual; and

19                  reducing the plurality of public keys by the one public key.

1                   9.     The system of claim 6 wherein the sequence of  
2 electronic elements are public keys, and wherein the server if further  
3 configured to check, in response to a request from an individual, that the  
4 individual has a value uniquely and mathematically related to a one of the  
5 public keys; and

6                   if so, issue a certificate to the one individual.

1                   10.    A computer-implemented method, comprising:  
2                   receiving a plurality of public keys from a corresponding  
3 plurality of individuals, wherein each of the plurality of individuals have a  
4 private key corresponding to one of the plurality of public keys;  
5                   receiving a request for a certificate from one of the plurality  
6 of individuals having a one private key;  
7                   providing at least a subset of the plurality of public keys to the  
8 requesting individual;  
9                   receiving a shuffle of the plurality of public keys and a linear  
10 size proof of correctness for the shuffled public keys based on a scaled  
11 iterated logarithmic multiplication proof and a value corresponding to the  
12 one private key, wherein the value provides proof that the one individual  
13 has knowledge of the one private key without revealing the one private key;  
14                   checking the proof of correctness;  
15                   checking that the value is mathematically related to a one of  
16 the public keys that corresponds to the one private key;  
17                   issuing a certificate to the one individual; and  
18                   reducing the plurality of public keys by the one public key.

1                   11.    The method of claim 10 wherein the method further  
2 includes setting a value  $G$  to a subgroup operator  $g$  from an  $Z_p$  or elliptic  
3 curve group, wherein providing at least a subset of the plurality of public  
4 keys includes providing all of the then current public keys  $H$ .

1                   12.    The method of claim 10 wherein providing at least a  
2 subset of the plurality of public keys includes providing at least a subset of  
3 a plurality of public key pairs, wherein receiving a shuffle of the plurality of  
4 public keys includes receiving a shuffle of a true subset of the plurality of  
5 public key pairs as selected by the one individual.

1                   13.    The method of claim 10, further comprising:  
 2                   receiving from each of a plurality of authorities, in sequence,  
 3    a shuffled set of the plurality of public keys H' based on a secret  
 4    cryptographic shuffle operation performed on at least a subset of the  
 5    plurality of public keys to produce the shuffled set of the plurality of public  
 6    keys H';  
 7                   receiving from each of a plurality of authorities, in sequence,  
 8    a verification transcript of the cryptographic shuffle operation; and  
 9                   verifying a correctness of the cryptographic shuffle operation  
 10   based on the verification transcript; and if verified, then setting at least a  
 11   subset of the plurality of public keys to H to H'.

1                   14.    The method of claim 10 , further comprising:  
 2                   at a time after receiving at least some of the plurality of public  
 3    keys, setting at least a subset of the then received plurality of public keys to  
 4    a received shuffled set of the plurality of public keys, wherein the shuffled  
 5    set of the plurality of public keys have been received from a third party.

1                   15.    The method of claim 10, further comprising:  
 2                   receiving the issued certificate from the one of the plurality of  
 3    individuals; and  
 4                   providing an electronic ballot to the one individual.

1                   16.    The method of claim 10 wherein issuing a certificate  
 2    includes digitally signing the received request to produce a public key  
 3    infrastructure ("PKI") certificate.

1                   17.    The method of claim 10 , further comprising:

2 receiving issued certificates from at least some of the plurality  
3 of individuals and providing initial electronic ballots in response thereto;  
4 and  
5 receiving unencrypted voted ballots from the at least some of  
6 the plurality of individuals.

1 18. A computer-implemented cryptographic method  
2 between a prover computer and a verifier computer, the method comprising:  
3 selecting a subgroup generator  $g$  selected from a group  $G$ ;  
4 secretly generating a prover key  $c$ , and a commitment value  $C$   
5 based on the subgroup generator  $g$ ;  
6 secretly establishing a cryptographic relationship between first  
7 and second sequences of elements;  
8 providing to the verifier computer the commitment  $C$  and the  
9 first and second sequences of elements, but not the cryptographic  
10 relationship;  
11 computing a series of proof values based on the cryptographic  
12 relationship; and  
13 providing the series of computed proof values to the verifier  
14 computer as a non-interactive proof of knowledge that the prover computer  
15 has access to the cryptographic relationship without revealing the  
16 cryptographic relationship to the verifier computer.

1 19. The method of claim 18 wherein at least the second  
2 sequence of elements is a sequence of encrypted ballots, wherein each  
3 ballot is encrypted using  $Z_p$  or elliptic curve groups;  
4 wherein the first and second sequences of elements are  
5 respectively

$$(X_1, \dots, X_k) \text{ and } (Y_1, \dots, Y_k)$$



6 wherein the first and second sequence of elements have the  
7 cryptographic relationship

8 and wherein computing and providing the series of proof

$$(g^{u_1}, \dots, g^{u_k}) = (X_1, \dots, X_k)$$
$$(g^{v_1}, \dots, g^{v_k}) = (Y_1, \dots, Y_k) \text{ and where}$$

$$c^k \prod_{i=1}^k u_i = \prod_{i=1}^k v_i$$

9 values includes providing Chaum-Pedersen proofs based on:

for each  $0 \leq i \leq k$  generate random  $r_i$

$$R_i = g^{r_i}$$

for each  $1 \leq i \leq k$   $w_i = r_i u_i / r_{i-1}$

$$W_i = g^{w_i}$$

$$z_i = w_i / v_i$$

$$Z_i = g^{z_i}$$

10

11 wherein the Chaum-Pedersen proofs provided to the verifier computer are  
12 of a form:

$$(R_{i-1}, X_i, R_i, W_i) \text{ and } (Y_i, C, W_i, Z_i).$$

13

- 1 20. The method of claim 18, further comprising:  
2 permuting the first sequence of elements to produce the  
3 second sequence of elements based on a cryptographic transformation;  
4 receiving a randomly generated value t from the verifier  
5 computer;  
6 secretly generating a value T based on the received value t and  
7 the subgroup generator, and secretly generating a value S based on the  
8 received value t and the prover key c; and

9 wherein computing and providing to the verifier computer the  
10 series of proof values includes providing a series of values based on the  
11 cryptographic transformation as a proof of knowledge that the prover  
12 computer has access to how the cryptographic transformation permuted the  
13 first sequence of elements to produce the second sequence of elements  
14 without revealing the cryptographic transformation to the verifier computer.

1 21. The method of claim 18, further comprising:  
2 permuting the first sequence of elements to produce the  
3 second sequence of elements based on a cryptographic transformation in a  
4 form of

$$(g^{u_1}, \dots, g^{u_k}) = (X_1, \dots, X_k)$$

$$(g^{cu_{\pi(1)}}, \dots, g^{cu_{\pi(k)}}) = (Y_1, \dots, Y_k)$$

5 receiving a randomly generated value  $t$  from the verifier  
6 computer;

7 secretly generating a value  $T$  based on raising the subgroup  
8 generator  $g$  to the received value  $t$ , and secretly generating a value  $S$  based  
9 on raising the value  $T$  to the prover key  $c$ ; and

10 wherein computing and providing to the verifier computer the  
11 series of proof values includes providing a series of values based on the  
12 cryptographic transformation in a form of:

$$U_i = X_i / T$$

$$V_i = Y_i / S$$

13 as a proof of knowledge that the prover computer has access to how the  
14 cryptographic transformation permuted the first sequence of element to  
15 provide the second sequence of elements without revealing the  
16 cryptographic transformation to the verifier computer.

1                   22.    The method of claim 18, further comprising:  
2                   receiving the first sequence of elements as a set of elements  
3 that have previously been permuted in a manner unknown to the prover  
4 computer;  
5                   receiving a series of randomly generated values  $e_i$  from the  
6 verifier computer;  
7                   secretly generating a series of values  $U_i$  based on a secret  
8 cryptographic transformation that employs the received series of values  $e_i$   
9 and secretly generated values  
10                    $\bar{u}_i$   
11                   permuting the second sequence of elements with respect to the  
12 first sequence of elements based on the series of values  $U_i$  and a secret value  
13  $d$ ; and  
14                   wherein computing and providing to the verifier computer the  
15 series of proof values includes providing the resulting values  $U_i$  and  
16 providing a series of proof values based on the cryptographic transformation  
17 as a proof of knowledge that the prover computer has access to how the  
18 cryptographic transformation permuted the first sequence of element to  
19 provide the second sequence of elements without revealing the  
cryptographic transformation to the verifier computer.

1                   23.    The method of claim 18, further comprising:  
2                   receiving the first sequence of elements as a set of elements  
3 that have previously been permuted in a manner unknown to the prover  
4 computer;  
5                   receiving a series of randomly generated values  $e_i$  from the  
6 verifier computer;  
7                   secretly generating a series of values  $U_i$  based on a secret  
8 cryptographic transformation of a form

$$u_i = \bar{u}_i + e_i = \log_g U_i$$

9

10                   permuting the second sequence of elements with respect to the  
11 first sequence of elements based on the series of values  $U_i$  and a secret value  
12  $d$  based on the following operations

$$(V_1, \dots, V_k) = (U_{\pi(1)}^d, \dots, U_{\pi(k)}^d)$$

$$D = g^d$$

$$v_i = \log_g V_i$$

$$A_i = X_i^{v_i}$$

$$B_i = Y_i^{u_i}$$

13

14 and wherein computing and providing to the verifier the series of proof  
15 values includes providing the resulting values  $U_i$ ,

$$A = \prod_{i=1}^k A_i$$

$$B = \prod_{i=1}^k B_i$$

16 and for  $1 \leq i \leq k$ , providing a series of proof Chaum-Pedersen of a form

17

$$(g, V_i, X_i, A_i) \text{ and } (g, U_i, Y_i, B_i)$$

18 and a Chaum-Pedersen proof for  $(D, A, C, B)$  as a proof of knowledge that  
19 the prover computer has access to how the cryptographic transformation  
20 permuted the first sequence of element to provide the second sequence of  
21 elements without revealing the cryptographic transformation to the verifier  
22 computer.

1                   24. The method of claim 23, further comprising repeating  
2 the receiving the first sequence of elements, receiving a series of randomly  
3 generated values, secretly generating a series of values, and permuting the

4 second sequence of elements for  $l$ -tuple of elements in the first sequence of  
5 elements.

1 25. The method of claim 22 wherein receiving the first  
2 sequence of elements includes receiving a subset of a set of identifying  
3 elements, wherein each identifying element in the set corresponds to an  
4 individual, and wherein the method further comprises:

5 receiving an anonymous certificate if the verifying computer  
6 verifies the series of proofs.

1 26. The method of claim 18 wherein the group  $G$  is  $Z_p$ .

1 27. The method of claim 18 wherein the group  $G$  is an  
2 elliptic curve group.

1 28. A computer-readable medium whose contents provide  
2 instructions, when implemented by a computer, perform a shuffling of a  
3 sequence of electronic data elements, comprising:

4 receive the sequence of electronic data elements;  
5 apply a secret, one-way cryptographic transformation using at  
6 least a first secret key to anonymously permute the sequence of electronic  
7 data elements and produce a first shuffled sequence of electronic data  
8 elements; and

9 generate a first linear size, non-interactive proof of  
10 correctness for the first shuffled sequence of electronic data elements based  
11 on a scaled iterated logarithmic multiplication proof.



1                   33.    The computer-readable medium of claim 28 wherein  
2   the computer-readable medium is a memory of a computer system.

1                   34.    The computer-readable medium of claim 28 wherein  
2   the computer-readable medium is an Internet connection link to a voting  
3   authority server computer.

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1                   35.    In a cryptographic method, a transmitted signal for use by a computer,  
2 comprising:  
3                   a shuffled sequence of electronic data elements representing individual data  
4 files, wherein a one-way cryptographic transformation using at least a first secret key  
5 anonymously permuted an input sequence of electronic data elements to produce the shuffled  
6 sequence of electronic data elements, and  
7                   a linear size proof of correctness for the shuffled sequence of electronic data  
8 elements based on a scaled iterated logarithmic multiplication proof.

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